Iranian Mathematical Society

Second Day

40th Iranian Mathematics Competition

25 August 2016

Iran University of Science & Technology

7. Suppose that A and B are two 2×2 matrices with real enteries, such that AB is a linear combiantion of I, A and B. Show that BA is also a linear combination of I, A and B.
8. For every five points on the surface of a sphere, show that there exists a closed hemi-sphere including at least four points of them. (Hint: A closed hemi-sphere includes its boundary)
9. Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ be three sequences of non-negative real numbers. Suppose that $a_{n+1} \leq a_n - b_n + c_n$ for all $n \in \mathbb{N}$ and the series $\sum_{n=1}^{+\infty} c_n$ converges. Prove that the sequence $\{a_n\}$ converges too.
10. Show that there is no continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f(f(x)) = \cos x$, for all $x \in \mathbb{R}$.
11. Show that $2^n - 1$ does not divide $3^n - 1$ for all integer $n > 1$.
12. Suppose that G is a group with finitely many non-Abelian subgroups. Show that each infinite subgroup of G is normal.